LCOL

A sum of €3000 is invested in a five-year government bond with an annual equivalent rate (AER) of 3%. Find the value of the investment when it matures in five years' time.

$F = P(1+i)^t$

$F = 3000(1+.03)^5$

$F = €3477.82$

A different investment bond gives 15% interest after 6 years. Calculate the AER for this bond.

$1.15 = (1+x)^6$

$\sqrt[6]{1.15} = 1+x$

$1.0236 = 1+x$

$x = .0236$

$AER = 2.4\%$

LCOL

A machine cost €25,650 depreciates to a scrap value of €500 in 10 years. Calculate:

(i) the annual rate of depreciation.

$$F = P(1-i)^t$$

$$500 = 25650(1-x)^{10}$$

$$\frac{500}{25650} = (1-x)^{10}$$

$$0.6745 = 1-x$$

$$x = 0.32549$$

$$\text{Rate} = 32.5\%$$

(ii) The value of the machine at the end of the sixth year.

$$F = P(1-i)^t$$

$$x = 25650(1-.32549)^6$$

$$x = €2415.56$$

$$25650(1-.325)^6$$

$$2426.10731$$

$$x = €2426.11$$

NB accept either calculation for full credit

LCOL

A firm estimates that office equipment depreciates in value by 40% in its first year of use. During the second year it depreciates by 25% of its value at the beginning of that year. Thereafter, for each year, it depreciates by 10% of its value at the beginning of the year

Calculate:

(i) the value after eight years of equipment costing €550 new.

$F_A = P(1-i)^t$

$F_A = 550(1-0.4)^1$

$= €330$

$\Rightarrow$

$F_B = 330(1-0.25)^1$

$= €247.50$

$F_C = 247.50(1-0.1)^6$

$= 131.53$

Ans €131.53

(ii) the value when new of equipment valued at €100 after five years of use.

$F_A = P(1-0.4)^1$

$= P(0.6)$

$F_B = [P(0.6)](1-0.25)$

$= P(0.6)(0.75)$

$F_C = [P(0.6)(0.75)](1-0.1)^3$

$100 = P(0.6)(0.75)(0.729)$

$100 = P(0.32805)$

$\frac{100}{0.32805} = P \longrightarrow$ Ans €304.83

LCHL Q. Eamon and Sile have just had their first child, Donal. They are planning for his education in eighteen years' time. First, they calculate how much they would like to have in the education fund when Donal is eighteen. Then, they calculate how much they need to invest in order to achieve this. They assume that, in the long run, money can be invested at an inflation-adjusted annual rate of 2%. Your answers throughout this question should therefore be based on a 2% annual growth rate.

(a) Write down the present value of a future payment of €5,000 in one years' time.

$$P = \frac{5000}{1 \cdot 02}$$

$$= €4,901 \cdot 96$$

(b) Write down, in terms of *t*, the present value of a future payment of €5,000 in *t* years' time.

$$P = \frac{5000}{(1 \cdot 02)^t}$$

(c) Eamon and Sile want to have a fund that could, from the date of his eighteenth birthday, give Donal a payment of €5,000 at the start of each year for 5 years. Show how to use the sum of a geometric series to calculate the value on the date of his eighteenth birthday of the fund required.

Present value of each €5000

$$\frac{5000}{(1.02)^0} + \frac{5000}{(1.02)^1} + \frac{5000}{(1.02)^2} + \frac{5000}{(1.02)^3} + \frac{5000}{(1.02)^4}$$

5000 4901.96 4805.84 4711.61 4619.23

total = € 24038.64

(d) Eamon and Sile plan to invest a fixed amount of money every month in order to generate the fund calculated in part (c). Donal's eighteenth birthday is 18 x 12 = 216 months away.

(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 2%.

$1.02 = (1+i)^{12}$ $\quad (1.02)^{\frac{1}{12}} = 1+i$

$(1.02)^{\frac{1}{12}} - 1 = i \rightarrow 0.0016515$ $\quad$ Ans 0.1652%

(ii) Write down, in terms of *n* and *P*, the value on the maturity date of an education plan of €*P* made *n* months before that date.

$F = P(1+i)^n = P(1+0.001652)^n$

(iii) If Eamon and Sile make 216 equal monthly payments of €*P* from now until Donal's eighteenth birthday, what value of *P* will give the fund he requires?

$€24038.64 = P(1+i)^1 + P(1+i)^2 + \dots + P(1+i)^{216}$

$= P(1.001652)^1 + P(1.001652)^2 + \dots + P(1.001652)^{216}$

$S_n = \dfrac{a(r^n-1)}{r-1}$ $\quad$ with $a = P(1.001652)$

$r = 1.001652$

$n = 216$

$S_{216} = \dfrac{P(1.001652)(1.001652^{216}-1)}{(1.001652-1)} = 24038.64$

$P[.42908] = 39.71183$

$P = €92.55$

(e) If Eamon and Sile wait for ten years before starting Donal's education fund, how much will they then have to pay each month in order to generate the same education fund?

$$24038.64 = P(1+i)^1 + P(1+i)^2 + \cdots + P(1+i)^{96}$$

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \text{with} \quad a = P(1 + 0.001652)$$

$$r = 1.001652$$

$$n = 96$$

$$S_{96} = \frac{P(1.001652)(1.001652^{96} - 1)}{(1.001652 - 1)} = 24038.64$$

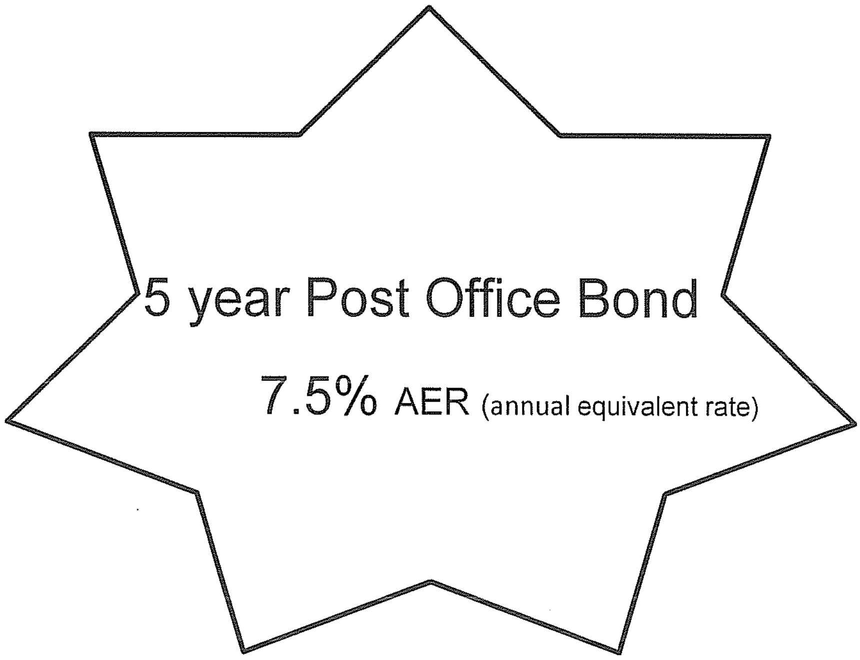
$$P[0.17199] = 39.71183$$

$$P = 230.8961$$

$$P = €230.90$$

LCFL

Alison has €6,000 to invest; she sees the following advert for a post office bond.



(i) If she invests her €6,000, what will the value of her investment be when it matures in five years' time.

Year 1 : 6,000 x 1.075 = 6450
Year 2 : 6450 x 1.075 = 6933.75
Year 3 : 6933.75 x 1.075 = 7453.78
Year 4 : 7453.78 x 1.075 = 8012.81
Year 5 : 8012.81 x 1.075 = €8613.78

(ii) How much interest will she have made?

8613.78 − 6,000 = €2613.78

(iii) Express her interest as a percentage of her original investment.

$\frac{2613.78}{6000}$ = 43.6%

LCHL

Carl sees a car on a car dealer's website. He clicks on the section saying "Finance this car" and finds out how much it would cost him to borrow €5,000.

The details are shown in the table:

		Fixed Rates		
Over 5 years	APR	Monthly Repayment	Total Repayable	Total Cost of Credit
€5000	14.9%	A	B	C

(i) Calculate the values **A** and **B** and **C.**

Monthly Rate $\rightarrow \sqrt[12]{1.149} = 1.011641575$

Present value of all Repayments = €5000

$$5000 = \frac{A}{\sqrt[12]{1.149}} + \frac{A}{\left(\sqrt[12]{1.149}\right)^2} + \cdots + \frac{A}{\left(\sqrt[12]{1.149}\right)^{60}}$$

$$= S_n \text{ of G.P. with } a = \frac{A}{\sqrt[12]{1.149}}\ ;\ r = \frac{1}{\sqrt[12]{1.149}}\ ;\ n = 60$$

$$\therefore\ 5000 = \frac{\frac{A}{\sqrt[12]{1.149}}\left(1 - \left[\frac{1}{\sqrt[12]{1.149}}\right]^{60}\right)}{1 - \frac{1}{\sqrt[12]{1.149}}}$$

<u>€116.26</u> = A

B = 116.26 × 60 = <u>€6975.60</u>

OR use the formula on p. 31 for amortization with

$i = \sqrt[12]{1.149} - 1$

$P = 5000$

$t = 60$

C = €6976.50 − €5000 = <u>€1975.60</u>

LCHL

John purchases a house, which is financed with a 20 year loan of €200,000 at a rate of 3% APR. On the property website where he saw the ad for the house, the mortgage calculator showed the following repayments:

Loan Information

Loan Amount (€):	200000	
APR	3	%
Repayment Term:	20	years

Calculate

Results: Mortgage Affordability Information

Total Monthly Payment: **A**

(a) Find the value of the monthly repayment **A.**

Monthly interest rate $= \sqrt[12]{1.03} = 1.00246627$

$i = 0.00246627 \quad P = 200{,}000 \quad t = 240$

$$A = 200{,}000 \left[\frac{0.00246627(1.00246627)^{240}}{(1.00246627)^{240} - 1} \right]$$

$$A = \frac{0.004454358}{0.806111333}(200{,}000)$$

$A = €1105.15$

LCHL

Tom wants to buy a car in three years' time. He estimates the car will cost €10,000 and so he decides to put a certain amount of money into a special savings deposit account that pays 4% AER compounded monthly. If this is to give him €10,000 in three years' time how much would he need to save each month?

Present value of €10,000

$$P = \frac{F}{(1+i)^t} \qquad P = \frac{10{,}000}{(1\cdot04)^3}$$

$$P = €8889.93$$

Monthly rate $= \sqrt[12]{1.04}$

$$A = \frac{8889.93\,(1.00327374)^{36}(0.00327374)}{(1.00327374)^{36}-1}$$

$A = €262.18$ every month.

LCHL

Mary deposits €500 each month in her savings account. If the account earns 2.5% AER compounded monthly, how much will she have in 5 years?

monthly interest rate : $(1.025)^{\frac{1}{12}} = 1.0020598836$

5 yrs $\Rightarrow$ 60 months $\qquad S_n = \dfrac{a(r^n-1)}{r-1}$

$$S_{60} = 500(1.0020598836)^1 + 500(1.0020598836)^2 + \dots + \dots 500(1.0020598836)^{60}$$

$$= \frac{500(1.0020598836)\left[1.0020598836^{60} - 1\right]}{0.0020598836}$$

$= 31963.44$

Ans €31963.44

LCHL

Sarah has €30,000 in a deposit account that pays 6% AER. Sarah believes that after 15 years this investment will be worth treble its present value. Is this true? If not, find the amount of years correct to the nearest year for this investment to treble.

$$30{,}000(1.06)^{15} = €71{,}896.75$$

No this is not true.

$$30{,}000 = \frac{90{,}000}{(1.06)^t}$$

$$30{,}000(1.06)^t = 90{,}000$$

$$(1.06)^t = 3$$

$$t = \frac{\log 3}{\log 1.06}$$

$$t = 18.5$$

$t = 19$ years.

LCFL

€1500 is invested for 5 years in a savings account which pays compound interest at a rate of 5% per annum provided the €1500 is left invested over the five-year period.

(i) How much money, to the nearest euro, will be in the savings account at the end of the five years if no money is withdrawn from the account?

(ii) If the interest is withdrawn at the end of each year, but the €1500 is left invested, what will be the difference in the total interest earned on the account over the five years?

(i) $A = 1500 \times (1.05)^5 = 1914.42$ Ans €1914

(ii) Interest $= 1500 \times 0.05 = 75$

$75 \times 5 = 375$

$414 - 375 = 39$ Ans €39

$1914 - 1500 = 414$

LCFL

Mary takes out a loan of €7500 from her credit union and agrees to pay back €2000 per year until the loan and interest is paid off. At the end of each year, before the repayments are made, the credit union charges interest at the rate of 7% per annum on the outstanding amount at the start of that year.

(i) Complete the table below showing the balance owed at the start of each year and the amount of interest (to the nearest euro) charged at the end of that year. Assume that the €2000 is paid each year as planned.

Start of Year	**Balance owed**	**Interest due**	**Total Due**
1	€7500	€525	€8025
2	€6025	€422	€6447
3	€4447	€311	€4758
4	€2758	€193	€2951
5	€ 951	€ 67	€1018

LCOL

The National Treasury Management Agency (NTMA) offers a National Solidarity Bond which earns cumulative interest of 1% for each year the bond is kept. This interest is taxable at 27%. If the bond is kept for at least 5 years, a tax-free lump sum is also paid, as shown in the table

10 Year National Solidarity Bond

At the end of Year	Cumulative 1% Annual Interest[1]		Bonus Tax Free Lump Sum
1	1%	+	0%
2	2%	+	0%
3	3%	+	0%
4	4%	+	0%
5	5%	+	10%
6	6%	+	10%
7	7%	+	22%
8	8%	+	22%
9	9%	+	22%
10	10%	+	40%

(i) What will be the value of a €1000 bond if it is cashed in after 4 years, and the cumulative interest is taxed at 27%?

$4\% = €40$
$27\% = €10.80$ } €29.20 net

Ans €1029.20

(ii) €1000 is invested in this bond and the bond is cashed in at the end of the 10 years. If the cumulative interest is taxed at the rate of 27% and the tax-free bonus sum is paid as shown, what is the value of the bond after the 10 years

$10\% = €100$
$27\% = €27$ } €73 net interest

lump sum { $40\% = €400$ ⇒ $1000 + 400 + 73$

Ans €1,473 after 10 yrs