Strand 5 Resources – Leaving Certificate

In Strand 5, you extend your knowledge of patterns and relationships from Strands 3 and 4 and build on your experience of these strands in the junior cycle. You should now be able to make connections between coordinate geometry, algebra, functions and calculus.

The following pages contain activities related to functions and calculus. Try the concept of slope presentation first and as you work through the activities think about how these connect with other areas of the mathematics course.

Concepts in calculus

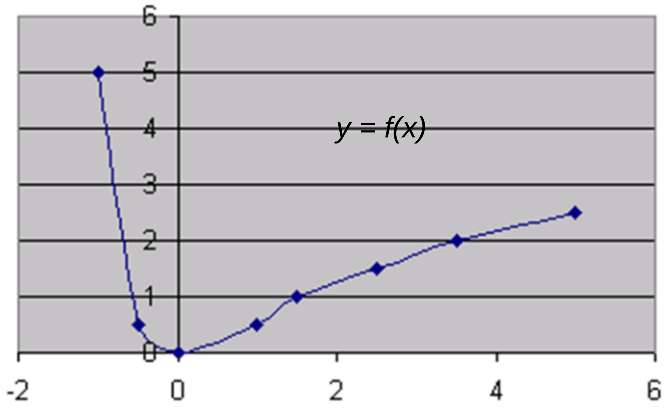
One of the aims of Project Maths is to help students develop conceptual understanding of mathematics. If you have conceptual understanding you will be able to

- generalise from particular examples
- apply and adapt ideas to new situations
- approach problems visually, numerically or algebraically and convert easily from one representation to another
- associate meaning with results
- connect old ideas with new ideas
- understand the limitations of an idea.

One can assess conceptual understanding by using problems/tasks that require you to do all of these things. Some problems/tasks related to functions are shown below. You may find these problems/tasks difficult and you may not be able to quickly see a solution strategy. Do not worry; these concept problems, by very definition, are not routine but designed to place you in new situations where you will have to apply and adapt what you have learned.
You may find it helpful to work through these questions with a study partner, The emphasis here is not on obtaining the correct answer but rather on making you think and discuss and on the reasoning and sense-making opportunities the problems afford you.

Examples of student work are included for a selection of the tasks, Try the tasks yourself before you look at other students' work. We invite you to **Compare, Examine, Discuss and Evaluate** the solution strategies provided.

Q The diagram shows the graph of the function $y = f(x)$ for $-1 \leq x \leq 5$. Approximate the x-value(s) for which



(a) $f'(x) = 0$

(b) $f'(x) < 0$

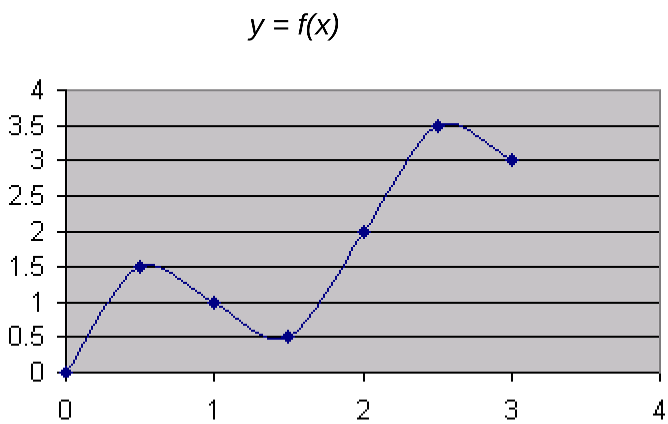
(c) $f'(x) > 0$

(d) $f''(x) < 0$

(e) $f''(x) > 0$

Q The diagram shows the graph of the function $y = f(x)$ for $0 \leq x \leq 4$.

Approximate the x-value(s) for which



(a) $f'(x) = 0$

(b) $f'(x) < 0$

(c) $f'(x) > 0$

(d) $f''(x) < 0$

(e) $f''(x) > 0$

What is unusual about these two questions? You are asked to examine f′(x) and f′′(x) of given functions. What does *f′(x)* mean*?* What does *f′′(x)* mean*?* How can you find *f′(x)* or *f′′(x)* of a function when you are only given the graph of the function?

This is the challenge in these task; they are *assessing* how well you understand the concept of the derivative. You will only be able to answer them if you fully understand this concept.

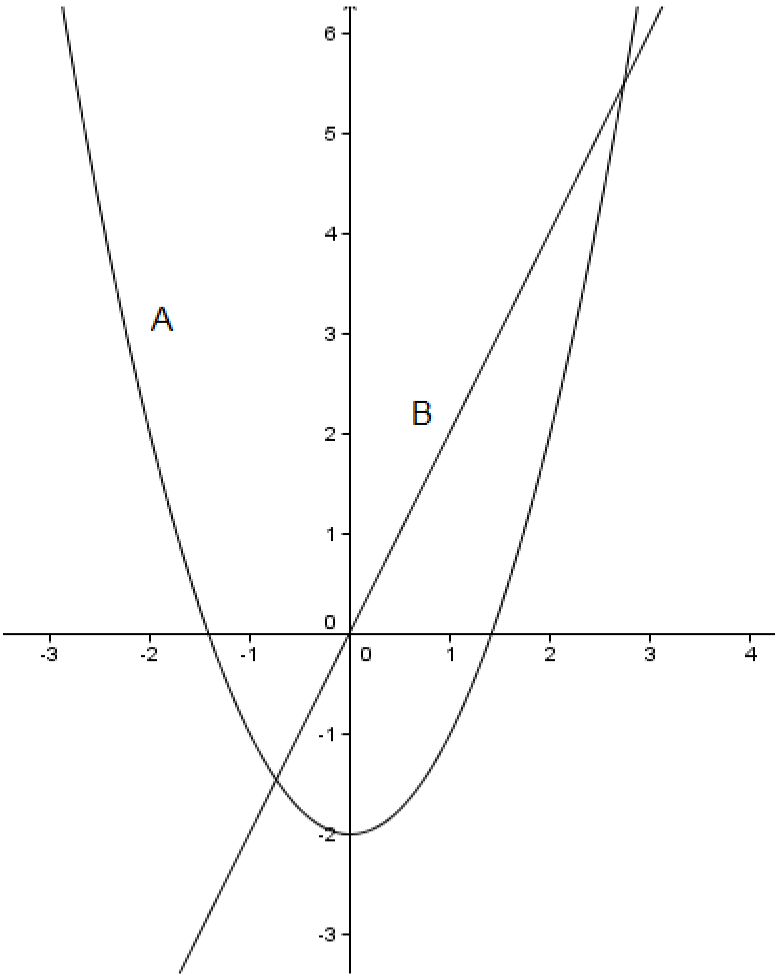
So; what if you don't fully understand the concept, how can you help yourself develop an understanding of the concept of the derivative?

- Work your way through *The concept of slope presentation*
- Read the document entitled *The Derivative: making sense of differentiation*
- Investigate with GeoGebra. You can download GeoGebra free at www.geogebra.org

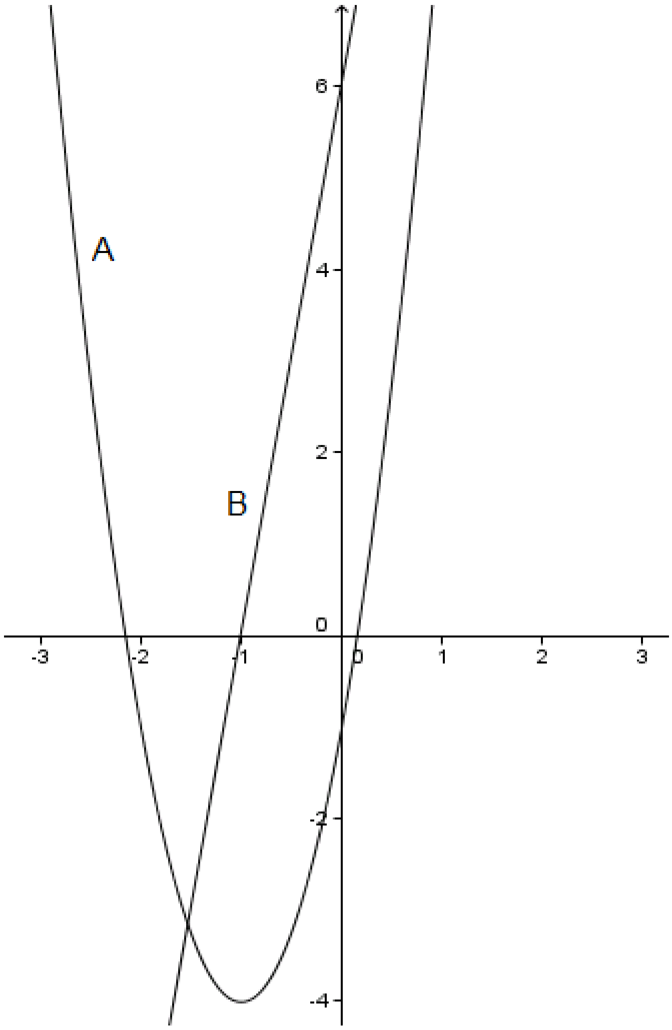
As you can see in order to answer these questions you need to have a lot of understanding. It is not a matter of simply getting the right answer; the question really does require you to show deep conceptual understanding. Now that you have a better understanding of the derivative can you extend this understanding to the second derivative?

Other things you need to consider about the questions are what does it mean

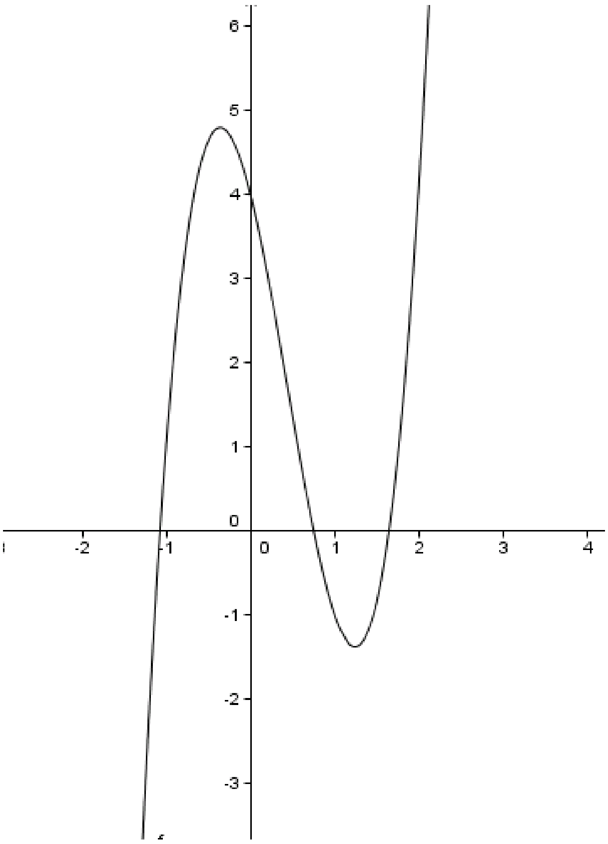
Q. The diagram shows the graph of a function and its derivative. Which is which? Give at least three reasons to support your choice.



Q. The diagram shows the graph of a function and its derivative. Which is which? Give at least two reasons to support your choice.

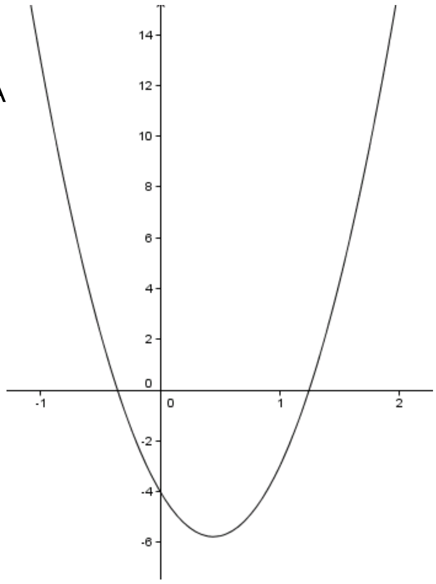


Q. The diagram shows the function f(x)

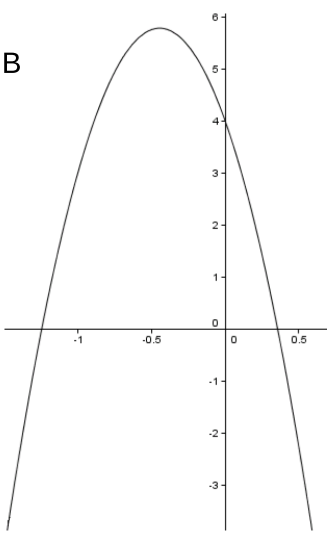


Which of the graphs below: A, B, C, D shows the derivative of *f(x)?* Give 3 reasons for your answer.

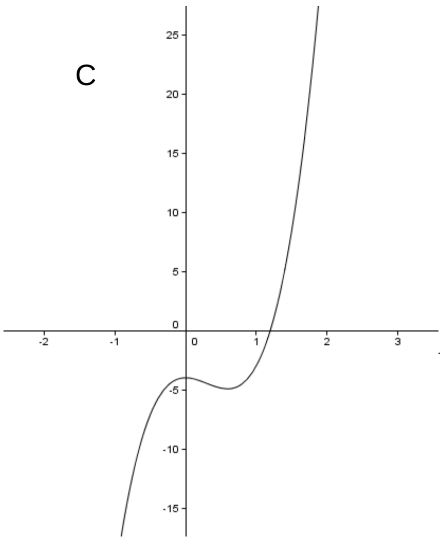
A



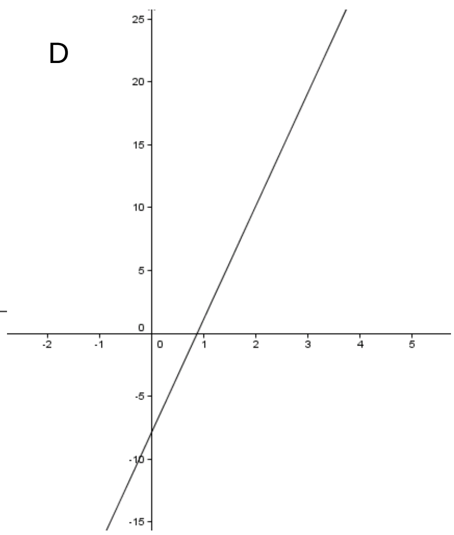
B



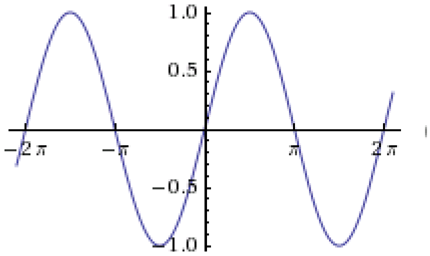
C



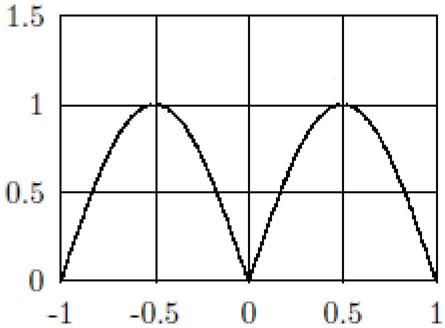
D



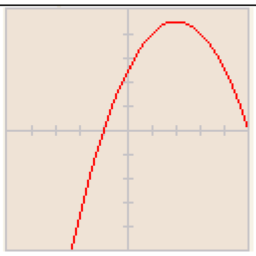
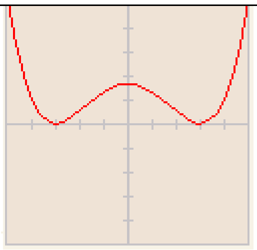
Q Sketch the derivative of the function shown

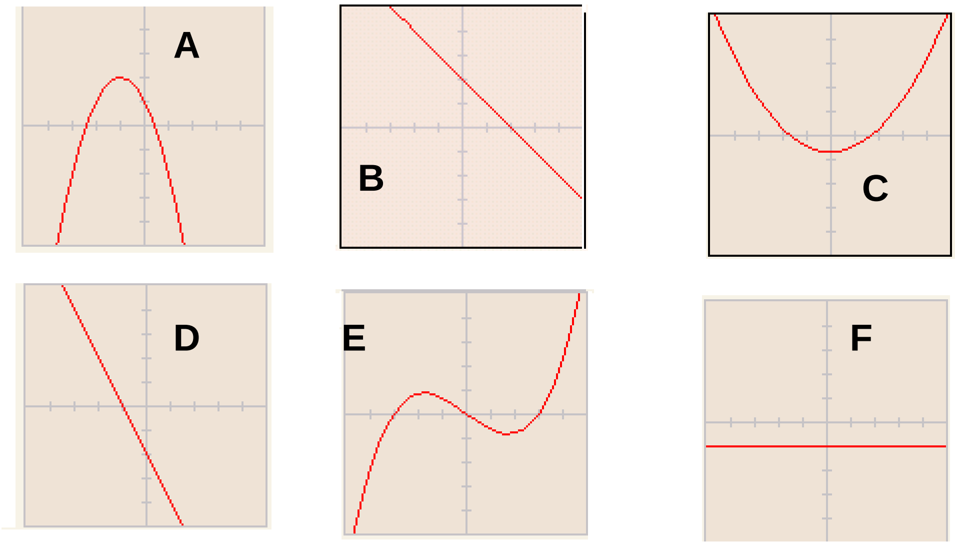


Q Consider the function y = h(x) whose graph is shown below. Find the critical points of h and determine intervals on which h′(x) is (i) positive and (ii) negative. Give reasons for your answer in each case.



Q The graphs of three functions are shown in the table. Six other graphs, labelled A, B, C, D, E and F, are shown below the table. Complete the table by inserting the appropriate letter in each of the empty cells.

Function	First Derivative	Second derivative
		
		
		



Q The table below gives the values of a function f and its first and second derivatives at selected values of x. Determine which row gives the data for f, which row gives the data for f′, and which row gives the data for f′′. Explain your reasoning.

x	0.00	0.33	0.66	1.00	1.33	1.66	2.00	2.33	2.66	3.00
A(x)	0.00	0.64	1.14	1.38	1.28	0.84	0.08	- 0.89	- 1.91	- 2.83
B(x)	0.00	0.11	0.41	0.84	1.30	1.66	1.82	1.69	1.22	0.42
C(x)	2.00	1.78	1.16	0.24	- 0.83	- 1.85	- 2.65	- 3.07	- 3.00	- 2.40

Q Suppose a car is driving on a straight road and that its velocity is positive for the first hour and then negative for the next 20 minutes. What can you conclude?

Q Suppose that the function m(t) gives the area of the lighted portion of the moon as seen from the front of your school. How can you express the times at which the moon is waxing (getting larger) and at which the moon is waning (getting smaller)?

Q. The table below gives the values of the function f at selected points. Find a reasonable approximation for f′ (1).

x	0.8	0.9	1.0	1.1	1.2
f (x)	1.67	1.85	2.03	2.21	2.38

Q For each of the following sentences identify

a function whose second derivative is being discussed

what is being said about the concavity of that function

The FTSE reacted to the latest report that the rate of inflation was slowing down.

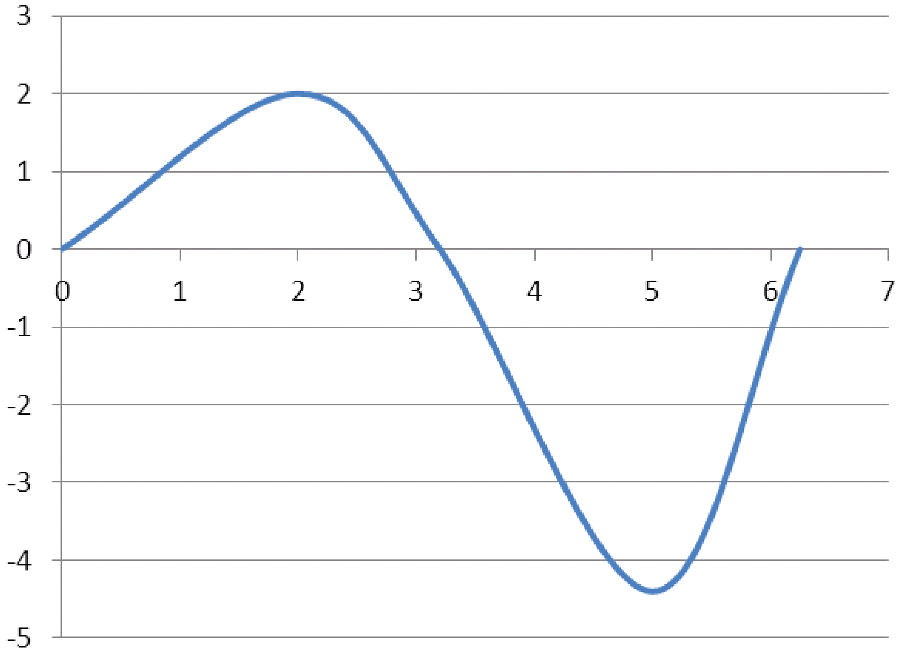
When he saw the light turn amber he hit the accelerator.

As the swine flu vaccination programme rolled out, the rate of new infections decreased dramatically.

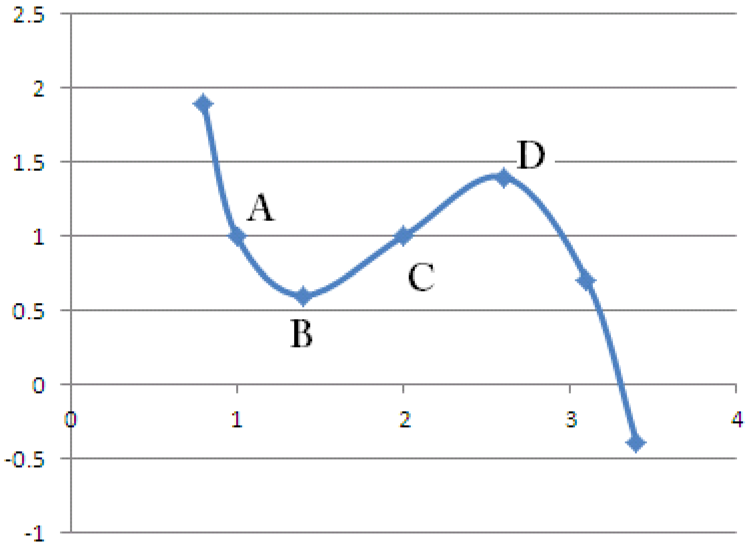
Q The graph of the function f is shown below. Referring to this graph, arrange the

following quantities in ascending order.

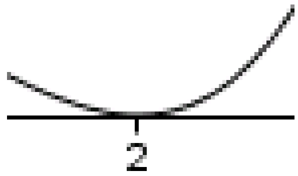
f′ (1), f′ (5), f (5), f (7), f′ (3)



Q. The graph shows a function y = f(x). At which labelled point(s) might it be possible that d2y/dx2 = dy/dx? Explain your reasoning.



Q A polynomial function p has degree 3. A part of its graph near the point (2, 0) is shown below.



Which one of the following could be the rule for the polynomial p?

Give reasons for your answer.

p(x) = x(x+2)2

p(x) = (x–3)3

p(x) = x2(x–2)

p(x) = (x–1)(x–2)2

p(x) = –x(x–2)2

Q. The table below gives the values of the function f at selected points.

Find a reasonable approximation for f′(1).

x	0.8	0.9	1.0	1.1	1.2
f(x)	1.67	1.85	2.03	2.21	2.38

Compare, Examine, Discuss and Evaluate

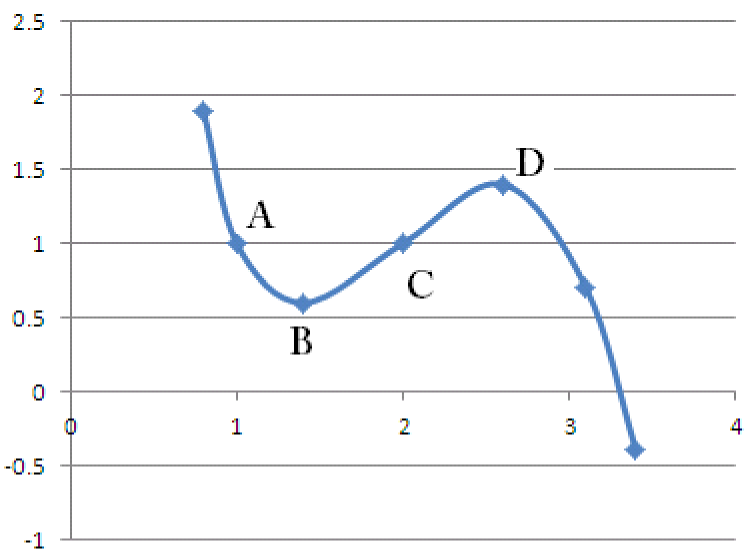
When I was first confronted with this question I thought: “I just can’t do it. I have never seen anything like this before.” Then my teacher told me to think about what I know about functions and their derivatives.

I remembered that a function can be represented in a table and a graph, and in a story or pattern. I can see the table and it looks like it is going up. I checked and I saw that it was going up by the same amount each time, 0.18. Now I know that this is a linear equation.

I felt more confident then and I went to Excel and put in the points and yes it is a straight line and I could see the slope was 0.18. Since the differentiation of the function is the slope I can say that a reasonable approximation for f′ (1) is 0.18 and it won’t change regardless of x because it is constant.

Q. At which labelled point(s) might it be possible that d2y/dx2 = dy/dx

Explain your reasoning.



Compare, Examine, Discuss and Evaluate

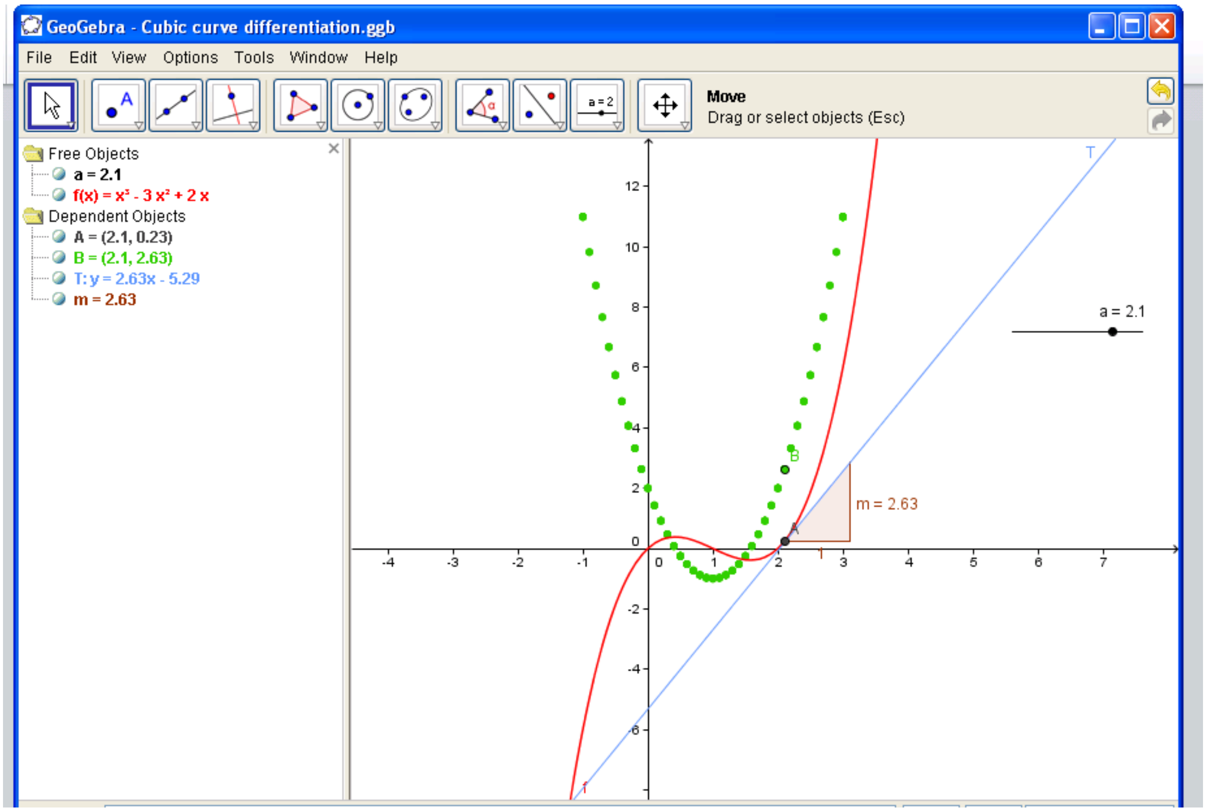
When my group first looked at this question we thought: right,…where do we start? I remembered what our teacher said ...make a list of things you know about functions – so we got to work.

This is a cubic function because it has a max and a min so that means the equation is in the form y = ax3+ bx2 + cx + d

When you differentiate this once you get a quadratic and when you differentiate it twice you get a linear one or a line

Josh remembered that when the graph is concave up the 2nd derivative is positive and when it's concave down the 2nd derivative is negative

But then I thought: oh no, maybe it's the other way round. So we thought: how can we decide? I said let's go to geogebra and find out. We were able to investigate lots with Geogebra . We discovered that Josh was right when he said when the graph is concave up the 2nd derivative is positive and when it's concave down it's negative. We could also see that when the function is increasing the first derivative is positive and when the function is decreasing the first derivative is negative



It took us a long time to come to this conclusion but Geogebra helped loads. The teacher told us to write about what we had learned in our learning journal. I thought that was a good idea 'cos we really had to think about it .We also learned that C was an interesting point as we couldn't decide if it was concave up or down. We argued both. The teacher told us that this is an interesting point because it is where the concavity changes; when the 2nd derivative changes from positive (concave up) to negative (concave down) it is called a point of inflection and the 2nd derivative is 0 there.

So we thought all the way to C the 2nd derivative is positive and all the way from C the 2nd derivative is negative. From A to B the function is decreasing and the 1st derivative is negative and from B to D the function is increasing so the 1st derivative is positive. In our graph C is a point of inflection ‘cos the 2nd derivative changes sign from positive to negative.

In the end we decided there were no labelled points where it was possible that the 1st and 2nd derivative were the same. At A the function is decreasing which means the 1st derivative is negative and concave up which means the 2nd derivative is positive. At B the function is a minimum which means the 1st derivative is 0. Also, it is concave up which means the 2nd derivative is positive.
C is the point of inflection which means the 2nd derivative = 0. It is not a max or min so the 1st derivative is not zero; it is a positive number ‘cos the function is increasing. D is a max and like B the 1st derivative is 0. But this time the 2nd derivative is negative ’cos it is concave down.

It took us ages to come to these conclusions but I really think I have it now.

We think the 1st and 2nd derivatives might be the same at points between B and C since the 1st and 2nd derivatives are both positive.

Having worked through this material you should have noticed that the questions required you to do a lot of thinking and it probably took you a long time to complete each question. You should have been busy thinking back to things you have done before and thinking how you could adapt those ideas to the new situations presented by the questions. You should have found that more than ever you were being required to attach meaning to your results. This is because they were concept questions which have been carefully designed to not only assess how well you have understood the concepts but also to give you an opportunity to reason and make sense of the new material in light of what you already know.

Think about how you approach the task; were you like the students whose work was **featured**? Were you lost in the beginning without a clue? The students whose work was featured mentioned a lot of strategies that helped them make sense of the tasks.

"My teacher said make a list of things you know about functions"

"Geogebra helped loads"

What do you know about functions? Could you spot a linear function in a context? In a table? In a graph? In a generalised equation? Could you spot a quadratic function in a context? In a table? In a graph? In a generalised equation? Can you tell when a function is increasing? Decreasing?

Could you spot an exponential function in a context? In a table? In a graph? In a generalised equation? What would the first derivative of each of these functions look like on a graph? In an equation? What would the second derivative look like?

Have a look at the concept of slope presentation.